**Graph Theory Fall 2020**

**Assignment 7**

**Due at 5:00 pm on Friday, November 13**

1. **Use an edge-counting argument to show that cannot be drawn on a 2-holed torus without edges crossing. Ingredients: For , you have . What would have to be? What is a lower bound on the total edge count since every region must be bounded by at least three edges?**

Given above for K9 is n = 9, m = . This means it will have 9 vertices and edges.

= .

Using Euler’s formula of the torus 🡪 v- e + f = 0

we have

**f = 27** i.e. number of faces.

**Since** each edge separates 2 faces, we have Kf = 2e

81 72. This is a clash or contradiction.

Therefore, will not be constructed on the torus.

1. **If we re-orient the arcs around the diagram from #1 so they all point clockwise, what is the resulting value of ?**

1. **In terms of , how many tournaments are there with the node set ? This is equivalent to asking for how many ways are there to orient the edges of with vertex set .**

To denote the many ways to orient the edges of with vertex set {1, 2, 3…, n}, we will find the equation for the number of edges.

We have number of edges.

Therefore, the number of ways to orient them will be 🡪

Substituting the value from the first equation will be, for .

1. **Let be fixed. Show that there are exactly two orientations of with vertex set that are strongly connected.**

A graph is strongly connected if there is a directed path from each vertex to every other vertex.

The clockwise and anti-clockwise orientation of  is as specified below,

0 🡪 1 🡪 2 🡪 … 🡪(n-2)🡪 (n-1) 🡪 0

0 🡪 (n-1) 🡪 (n-2) 🡪 … 🡪 2 🡪 1 🡪 0

Now, taking a different orientation, we will take a, b and c such that,

, where there is are paths from a to b and c to b. Whereas there are no paths from b to a or b to c.

Therefore, due to not all vertices having a directed path, graph not ‘strongly connected’.